7.2 (continued)

use Laplace transform for linear system

$$x' = x + 2y \qquad x(0)=1,\ y(0)=0$$
$$y' = 2x + y$$

transform each equation

$$\mathcal{L}\{x\} = X \qquad \mathcal{L}\{y\} = Y$$

$$\mathcal{L}\{x'\} = \mathcal{L}\{x + 2y\}$$

$$sX - x(0) = X + 2Y$$
$$sY - y(0) = 2X + Y$$

$$sX - X - 2Y = 1$$

$$\left.\begin{aligned}(s-1)X - 2Y &= 1\\ -2X + (s-1)Y &= 0\end{aligned}\right\} \quad \begin{bmatrix} s-1 & -2 \\ -2 & s-1 \end{bmatrix}\begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

the system can be solved in many ways

Cramer's rule is particularly useful

$$\underline{X} = \frac{\begin{vmatrix} 1 & -2 \\ 0 & s-1 \end{vmatrix}}{\begin{vmatrix} s-1 & -2 \\ -2 & s-1 \end{vmatrix}}$$

det. of coeff. matrix 1st column replaced by the right side

determinant of coeff. matrix

$$\underline{X} = \frac{s-1}{(s-1)^2-4} = \frac{s-1}{s^2-2s-3} = \frac{s-1}{(s-3)(s+1)} = \cdots$$

$$= \frac{1/2}{s-3} + \frac{1/2}{s+1}$$

$$x(t) = \frac{1}{2}e^{3t} + \frac{1}{2}e^{-t}$$

$$Y = \frac{\begin{vmatrix} s-1 & 1 \\ -2 & 0 \end{vmatrix}}{\begin{vmatrix} s-1 & -2 \\ -2 & s-1 \end{vmatrix}} = \frac{2}{(s-3)(s+1)} = \cdots \quad y(t) = \frac{1}{2}e^{3t} - \frac{1}{2}e^{-t}$$

7.3 Translation of Laplace Transform

we know $\mathcal{L}\{f(t)\} = F(s) = \int_0^\infty f(t)\, e^{-st}\, dt$

what happens to $f(t)$ if we do a translation in s domain

$F(s-a) \rightarrow f(t)$?

shift to the right by a

$$F(s) = \int_0^\infty f(t)\, e^{-st}\, dt$$

$$F(s-a) = \int_0^\infty f(t)\, e^{-(s-a)t}\, dt$$

$$\boxed{F(s-a) = \int_0^\infty \left[f(t)\, e^{at}\right] e^{-st}\, dt = \mathcal{L}\{f(t)\, e^{at}\}}$$

Translation Theorem (1)

from table : $\mathcal{L}\{1\} = \frac{1}{s} = F(s)$

$$F(s-a) = \frac{1}{s-a} = \mathcal{L}\{1 \cdot e^{at}\} = \mathcal{L}\{e^{at}\}$$

$$\mathcal{L}\{\sin(at)\} = \frac{a}{s^2+a^2} = F(s)$$

$$F(s-c) = \frac{a}{(s-c)^2+a^2} = \mathcal{L}\{e^{ct}\sin(at)\}$$

<u>example</u> $y'' - 2y' + 5y = 8e^t$

$y(0)=2, \quad y'(0)=4$

$$s^2Y - sy(0) - y'(0) - 2(sY - y(0)) + 5Y = \frac{8}{s-1}$$

$$(s^2-2s+5)Y = 2s + 4 - 4 + \frac{8}{s-1}$$

$$Y = \frac{2s}{s^2-2s+5} + \frac{8}{(s-1)(s^2-2s+5)}$$

$$= \frac{2s(s-1)+8}{(s-1)(s^2-2s+5)}$$

$$Y = \frac{2s^2 - 2s + 8}{(s-1)\left[(s-1)^2 + 4\right]}$$

$$= \frac{A}{s-1} + \frac{Bs + C}{(s-1)^2 + 4}$$

$$2s^2 - 2s + 8 = A\left[(s-1)^2 + 4\right] + (Bs + C)(s-1)$$

$$= A(s^2 - 2s + 1) + 4A + Bs^2 - Bs + Cs - C$$

$$2s^2 - 2s + 8 = (A+B)s^2 + (-2A - B + C)s + (5A - C)$$

$$A + B = 2$$

$$-2A - B + C = -2$$

$$5A - C = 8$$

$$4A = 8 \qquad A = 2, \; B = 0, \; C = 2$$

$$Y = \frac{2}{s-1} + \frac{2}{(s-1)^2 + 4}$$

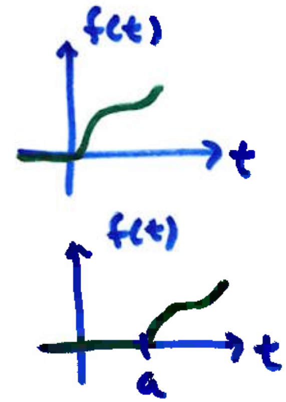
looks like $\frac{2}{s^2+4}$ shift by 1

$e^t \sin(2t)$

$$y = 2e^{t} + e^{t}\sin(2t)$$

1st Translation Theorem: $F(s-a) = \mathcal{L}\{e^{at} f(t)\}$

2nd Translation Theorem: $\mathcal{L}\{f(t-a)\} = ?$



$= 0$ for $t < a$

$$\mathcal{L}\{f(t-a)\} = \int_0^{\infty} f(t-a)\, e^{-st}\, dt$$

~~let τ = t + a~~

$$= \cancel{\int_a^{\infty} f(\tau)\, e^{-s(\tau+a)}}\, dt$$

$$= \int_a^{\infty} f(t-a)\, e^{-st}\, dt$$

delay in time